The Romer Model - some helpful tips
ECN 101 - Intermediate Macroeceonomic Theory
Professor Geromichalos, Winter 2012
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# 1 Equations

- $Y_t = A_t L_{yt}$ (1) Production Function
- $A_{t+1} - A_t = \bar{z} A_t L_{at}$ (2) Flow of Knowledge
- $L_{yt} + L_{at} = \overline{L}$ (3) How Labor is divided
- $L_{at} = \bar{l}\overline{L}$, $L_{yt} = (1-\bar{l})\overline{L}$ (4) A constant fraction of Labor is devoted to research (and production of goods)

# 2 Variables and Parameters

- $Y_t$ is output and will be given by the production function (Variable)
- $A_t$ is the stock of ideas or knowledge during time period $t$ (Variable)
- $L_{yt}$ $L_{at}$ are the number of laborers devoted to production of goods and production of ideas, respectively, in time $t$ (Variables)
- $\bar{l}$ is the fraction of labor devoted to production of goods (Parameter)
- $\bar{z}$ describes how efficient researchers are at producing new ideas (Parameter)
- $\overline{L}$ is the supply of workers (Parameter)
- $A_0$ is the initial level of ideas at $t = 0$ (Parameter)

# 3 Solving the Model

What are we solving for? The goal of this model is to explain why countries may experience sustained growth in the long run. Recall that the Solow Model says that there is no growth in the long run, after an economy reaches its steady state. Romer is trying to show that there can be growth in the long run (i.e. the rate of growth is not 0). **So, the goal is to solve for the growth rate of output and output per worker.**

To find the growth rate of output, let us first rewrite output in terms of parameters instead of variables: Substitute (4) into (1), so that (1) becomes,

$$Y_t = A_t(1-\bar{l})\overline{L}$$

Now, the growth rate of output is:

$$g(Y_t) = g(A_t) + g(1-\bar{l}) + g(\overline{L})$$

and since, $1-\bar{l}$ and $\overline{L}$ are constants (or parameters), then $g(1-\bar{l}) = 0$ and $g(\overline{L}) = 0$. That is, constants do not grow over time. Thus,

$$g(Y_t) = g(A_t)$$

We have found the growth rate of output! But wait, what is the growth rate of $A_t$?

Recall from Chapter 3 that the growth rate of a variable $X_t$ can be written as, $g(X_t) = \frac{X_{t+1} - X_t}{X_t}$ so thus $g(A_t) = \frac{A_{t+1} - A_t}{At}$

But how can we find the growth rate of $A_t$? Aha! Look at (2)! Divide (2) by $A_t$ on both sides and you will get

$$g(A_t) = \frac{A_{t+1} - A_t}{A_t} = \frac{\overline{z} A_t L_{at}}{A_t} = \overline{z} L_{at} = \overline{z}\overline{l}\overline{L}$$

where the last step follows from (4).

Yay! The growth rate of output is then given by

$$g(Y_t) = g(A_t) = \overline{z}\overline{l}\overline{L} \; (5)$$

So when $\overline{z}$, $\overline{l}$, and $\overline{L}$ are positive numbers, we have shown how an economy can achieve sustained growth.

# 4 Comparative statics

What happens when one parameter changes? For example consider if $\overline{z}$ decreases to $\overline{z_1}$ at time $t + 1$? How can we know what happens?

- First you should look at all the equations you have to see where $\overline{z}$ appears. Then you will know what variables are affected.

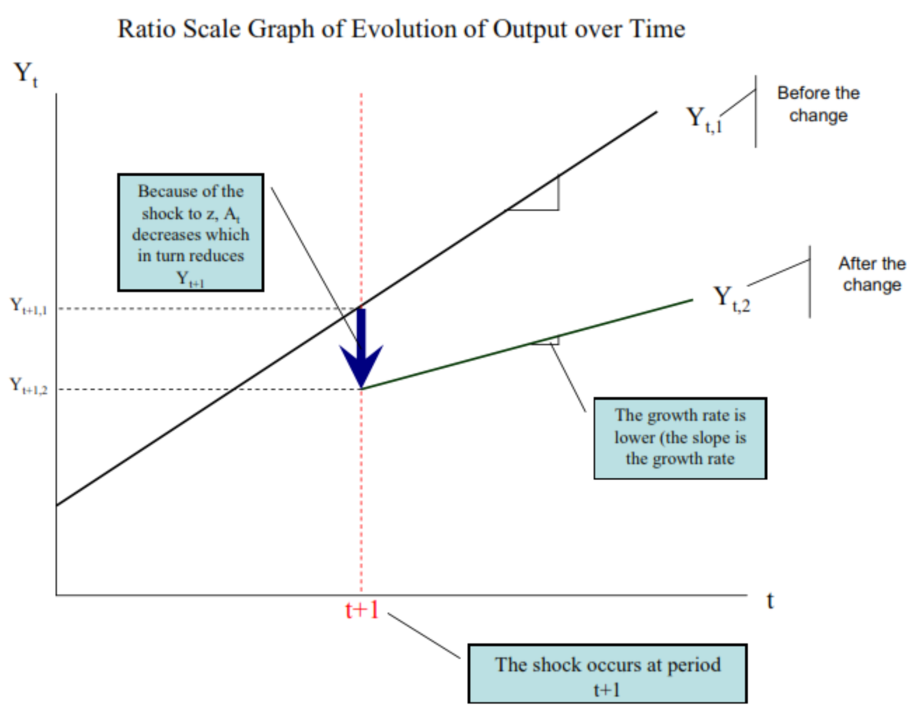
Looking back at all my equations, I see (2) and (5) depend directly on $\overline{z}$. So if we decrease to $\overline{z_1}$ then two things will decrease:

- $A_{t+1}$ will go down from (2). To see this more clearly just move $-A_t$ to the right hand side
- $g(Y_{t+1})$ and $g(A_{t+1})$ will go down

So changing $\overline{z}$ **caused** two other things to change. Now since two additional things have changed, does anything else change because of this?

- $Y_{t+1}$ will go down since $A_{t+1}$ went down, by equation (1)

So graphically this looks like



## 5 Growth rates revisited (a word of caution)

In the Romer model we solve for the growth rates of output and output per worker. One way to do this was shown above in section 3. There is a second way that some of you have been approaching this, which I would like to caution against. The second way goes like this:

We know our production function by (1) and from chapter 3 we also know that if $A_t$ is growing at a constant rate each year then we can write:

$$A_t = (1 + g(A_t))^t A_0 = (1 + \bar{z}\bar{l}\bar{L})^t A_0 \ (6)$$

where the last equality comes from the fact that we've solved for $g(A_t)$, see (5).

So we may want to rewrite our production function (1) by substituting for $A_t$ and $L_{yt}$ using (6) and (4):

$$Y_t = A_t L_{yt} = (1 + \bar{z}\bar{l}\bar{L})^t A_0 (1 - \bar{l})\bar{L} \ (7)$$

then to solve for the growth rate of output, you'd naturally take the growth rate of the right hand side of (7). This becomes

$$g(Y_t) = g[(1 + \bar{z}\bar{l}\bar{L})^t A_0 (1 - \bar{l})\bar{L}]$$

$$= g[(1 + \bar{z}\bar{l}\bar{L})^t] + g(A_0) + g[(1 - \bar{l})] + g(\bar{L})$$

we know the last three terms go to 0 because the growth rate of a constant is 0. So we are left with this final term

$$= g[(1 + \bar{z}\bar{l}\bar{L})^t]$$

then you may look back to chapter 3 and say "Aha! I've solved it! I know $g(X^a) = ag(X)$, and thus I'm done!"

But if you do this, you wil get an **incorrect** result. Suppose you proceed this way, then you will say:

$$= tg(1 + \bar{z}\bar{l}\bar{L})$$

which should equal to 0 because the growth rate of a constant is 0. **THIS IS INCORRECT!**

Why, you may ask, is this incorrect?

It is because this formula, $g(X^a) = ag(X)$, only applies when $a$ is a **constant**. ***But treating $t$ as a constant is where the mistake lies because $t$ is a variable that changes over time, NOT A CONSTANT.***

If you want to see how you can get the correct answer using (7), the correct way to proceed is to do $g(Y_t) = \frac{Y_{t+1} - Y_t}{Yt}$ using equation (7). This will give you the correct result. In fact try it yourselves and see!