A word of caution on calculating PDV

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What is PDV?

Let us revisit the concept of Present Discounted Value. Mathematically, we know the PDV of earnings in period t, which we denote w_t, is:

$$PDV_0 = \frac{w_t}{(1+R)^t} \tag{1}$$

Where the 0 subscript on PDV_0 means that the "Present" is period 0. Also remember R is the interest rate.

Intuitively what is this equation trying to say?

It is telling us that getting paid w_t dollars **in year t** is equivalent to getting paid PDV_0 dollars **today.**

This is becuase I can get PDV_0 dollars today (in period 0, or the "present"), and invest it at interest rate R. When I check how much money I have in period t, it will be equal to w_t.

To see this more clearly move $(1+R)^t$ in (1) to the left side, which gives:

$$PDV_0(1+R)^t = w_t \tag{2}$$

To get equation (2) let us consider slowly how your earnings evolve in each period. I give you PDV_0 dollars at time period 0. You take your money and deposit it in your savings account which earns interest rate R every year. How much will you have after one year (i.e. how much will you have at t=1)?

$$w_1 = PDV_0(1+R)^1 \tag{3}$$

In other words, after a year you will have in your account the initial amount (PDV_0) plus the interest earned.

After the second year (i.e. t=2) you will have

$$w_2 = w_1(1+R)^1 = PDV_0(1+R)(1+R) = PDV_0(1+R)^2 \tag{4}$$

Proceeding in this way we can see what we have after t years. It will be precisely equation (2).

Calculating PDV: CAUTION

You know the formula, but to apply it can be a little tricky. Let's review the formula for synthesizing a geometric series: If |a|<1, then

$$1 + a + a^2 + ... + a^n = \frac{1 - a^{n+1}}{1 - a} \tag{5}$$

Question: suppose your work career lasts 78 months. Each month you are paid your monthly salary of \$300. Your salary grows at a constant monthly rate of 1%. The interest rate is 4%. Calculate the PDV of your work career.

Answer:

First we need to translate this word problem into equations. My monthly salary is \$300. You will begin by defining the present period as t=0. Furthermore, you should keep in mind that the increment of time is months here. So t=0, 1, 2, 3, ... represent months.

Now, you must be careful here when it comes to time periods. Since our first period is 0, then our last period is t=77, NOT t=78. This is because of a counting issue. If you count t=0 as the first month then what you have is:

t	0	1	2	3	4	5	6	...	76	77
Month	1	2	3	4	5	6	7	...	77	78

Next, if my salary grows at a constant rate of 1% then this means:

$$w_t = w_0(1+g)^t, \text{ where } g = 0.01 \text{ and } w_t \text{ stands for salary in period t}$$

Also you should write down somewhere,

$$R = 0.04$$

Now to calculate the PDV of your work career, you need to sum the PDV of each period t earnings.

$PDV = \frac{w_0}{(1+R)^0} + \frac{w_1}{(1+R)^1} + \frac{w_2}{(1+R)^2} + \frac{w_3}{(1+R)^3} + ... + \frac{w_{77}}{(1+R)^{77}}$

now replace all the w_t with the above formula

$PDV = \frac{w_0}{(1+R)^0} + \frac{w_o(1+g)^1}{(1+R)^1} + \frac{w_0(1+g)^2}{(1+R)^2} + \frac{w_0(1+g)^3}{(1+R)^3} + ... + \frac{w_{77}(1+g)^{77}}{(1+R)^{77}}$

Now you can factor out w_0 and let $a = \frac{1+g}{1+R}$

$PDV = w_0(1 + a + a^2 + a^3 + ... + a^{77})$

Now it should look familiar. We can apply the geometric series formula:

$PDV = w_0(\frac{1-a^{78}}{1-a})$

Keep in mind it is really important to get n and n+1 right for your calculations to be correct. In order to get n and n+1 right, you need to remember that t=0,1,2,3..... is just a counter for the time periods, whether they be months, days, years, hours, etc. Importantly, always remember that t=0 is the FIRST time period. Thus if you work 55 hours, the last hour of work corresponds to t=54, because the first hour is t=0.

Lastly, when you are doing these PDV problems always try to write out the summation as I have done above in order to see patterns clearly. Where you will be able to apply the geometric series formula will depend on if you can clearly see the patterns.

Practice the problems in the book.

Best of Luck!